Chapter I

Fluid flow and viscosity

1.1 <u>Introduction</u>:

Properties of fluids, density, specific gravity, pressure, buoyancy, the flow of fluids, viscosity and surface tension, are important to biologists and medical scientists. Fluid studies can elaborate the blood flow in different blood vessels. In addition, vessel properties, wall elasticity, blood pressure, flow of blood and urine in and out of nephrons, …etc., can be thus studied.

Fluids are classified as follows:

Fluids

Pure fluids	Solutions	Suspension
e.g. water, oil, alcohol.	Solute + Solvent e.g. salt solutions, alkaline solutions	Small particles (physically distinguishable) + liquid e.g. blood

1.2 <u>Flow of Ideal Fluids</u>:

The main characteristics of ideal fluids are that they are incompressible, constant density, and have negligible internal frictional forces acting between the fluid layers i.e. they are non viscous.

1.2.3. Continuity Equation:

If the mass or volume rate of the flow is constant, the flow is said to be steady. Mass flow rate =*dm/dt*

$$\frac{dm}{dt} = \rho \frac{dV}{dt} \qquad \dots \quad (1.1)$$

This equation is applied to ideal fluids flowing in tubes that vary in cross section as that shown in fig. (1.1).

$$A_1 v_1 = A_2 v_2$$

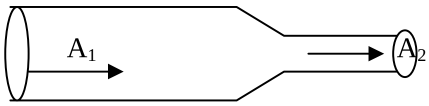


Fig. (1.1)

where A_1, A_2 are the cross sectional areas along the pipe and v_1, v_2 are the respective velocity values, ρ is the fluid density V is its volume, m is its mass.

The continuity equation shows that the velocity in smaller cross section is higher. Hence, the speed of blood in tiny capillaries should be higher than that in large arteries and veins, which contradicts reality. It is well known, that blood slows down in the blood capillaries surrounding the body cells, in order that gas and material exchange can easily take place. Thus the total cross sectional area of all the capillaries surrounding a certain organ must be much greater than the total area of the arteries feeding them.

$$\sum_{i=1}^{n} a_i > A_a$$
$$\therefore v_c < v_a \qquad \ldots (1.2)$$

where n is the number of branching blood capillaries each of cross sectional area, a, in which blood flows with velocity v_c, fed by an artery of cross sectional area A_a and of blood velocity v_a.

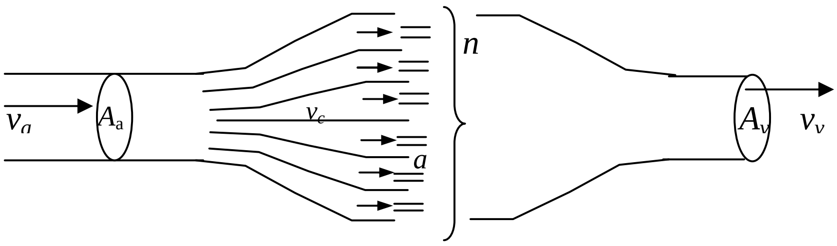


Fig.(1.2) The flow of blood from the wide artery A_a to n narrow capillaries, back to the vein A_v

1.2.2. Bernoulli's Equation:

Flow of fluids, in pipes for example, occurs when a pressure difference exists across the ends. The change in speed implies a change in the *K.E.* thus there must be work done on the fluid which is exerted by the pressure forces. Bernoulli's equation governs the variation of pressure, speed and elevation along the tube pipe

Work done on the fluid element of volume, *V*, to be transferred from point 1 to point 2 as shown in fig. (1.3) is expressed as:

$$\Delta W = \Delta P\ V$$

$$= (P_2 - P_1)\ V \qquad \ldots\ldots(1.3)$$

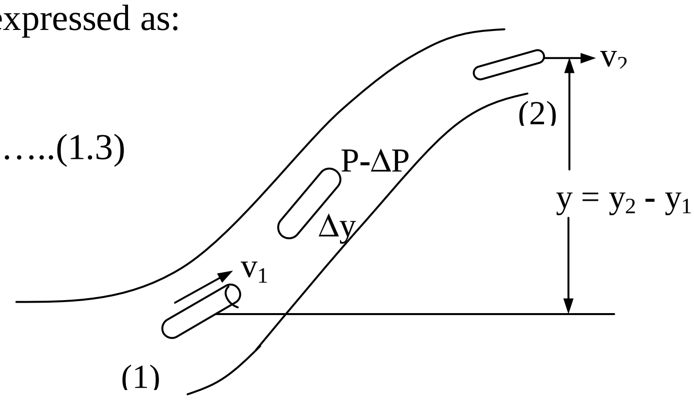


Fig. (1.3) Flow of a fluid element of volume V from point 1to point 2

$$E_1 = \frac{1}{2} m v_1^2 + m g y_1$$

$$E_2 = \frac{1}{2} m v_2^2 + m g y_2$$

$$E_2 - E_1 = \frac{1}{2} m \left(v_2^2 - v_1^2\right) + mg\left(y_2 - y_1\right) \qquad \ldots\ldots (1.4)$$

From the conservation of energy:

$$-\Delta W = E_2 - E_1$$

Work done per unit volume of the fluid:

$$P_1 - P_2 = \frac{1}{2} \rho\left(v_2^2 - v_1^2\right) + \rho g \left(y_2 - y_1\right)$$

Bernoulli's equation states that the total energy per unit volume, *H*, is constant. It is valid only for non-viscous fluid flow only.

$$H = P + \tfrac{1}{2}\rho v^2 + \rho g y = constant \qquad \ldots\ldots (1.5)$$

1.3 Pressure distribution in blood vessels:

The pressure waveform varies from one type of blood vessel to another.

For the blood flow in vessels, the total energy per unit volume, *H*, is not constant with time. *H* is a function of time, *H(t)*. *H(t)* decreases with time as blood loses energy along each cycle especially in the blood capillaries where the velocity decreases to be nearly zero. Why?

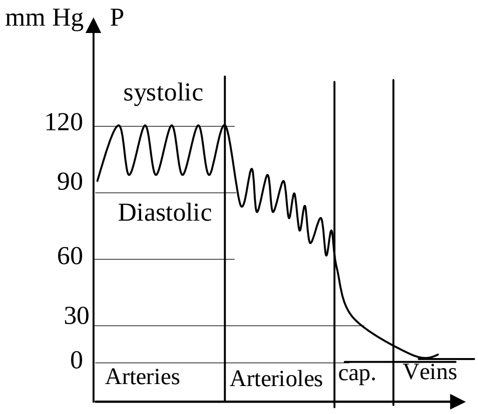


Fig.(1.4) Pressure variation in different blood vessels.

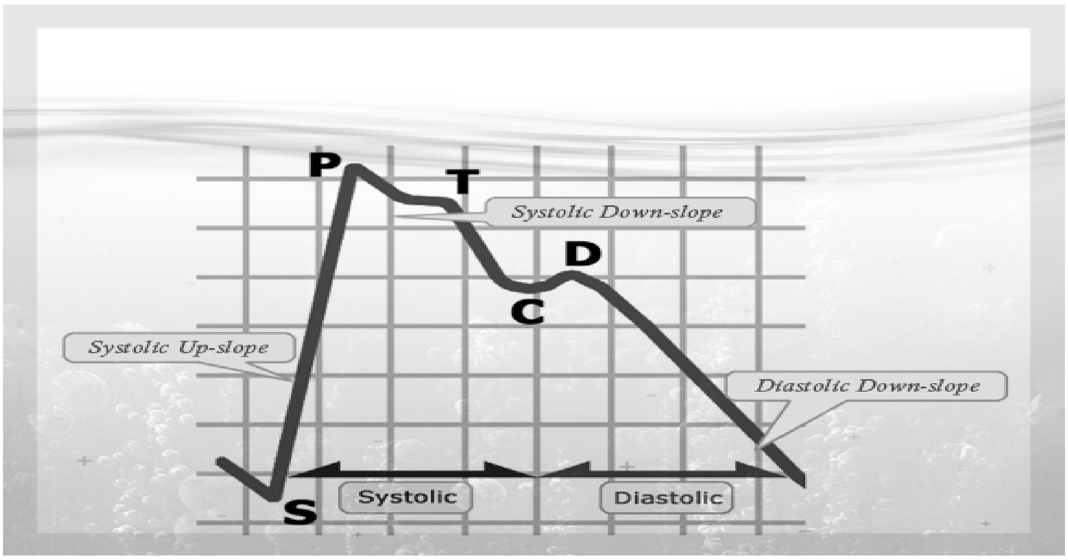


Fig.(1.5) One Arterial Pressure Pulse

The heart pumps the blood increasing H to its original level at the beginning of each blood cycle, back to the heart with minimum H, nearly zero. The variation of H follows the variation of blood pressure and the variation of velocity from one vessel to another. The pressure variation, at fixed time for an adult, is illustrated in fig (1.4). The arterial pressure pulse is shown in fig.(1.5). It is divided into a maximum part known as the "systolic pressure", equal to an average value of 120mmHg. (…..Pa) and a minimum arterial part, known as "diastolic pressure" , equal to an average value of 80mmHg. The pressure fluctuations are smoothed out as the blood returns back to the veins passing through the capillaries. The blood pressure, on reaching the capillaries is about 32mmHg and in veins is usually 12mm//Hg or less.

It is worthwhile to note that the blood pressure is determined by cardiac output, C_0, ($dV/dt = C_0$) and total peripheral resistance, R_p , according to the simple relationship, $P_0 = C_0 R_p$. Cardiac output can be changed, according to the body needs or activity, by increasing heart rate (pulse rate), or by increasing the force of contraction of heart ventricle, thus increasing the velocity. Also pressure variations within the circulatory system are nearly determined by peripheral resistance, R_p, that is explained later in this chapter.

Furthermore, the ***effective area of the heart valve*** is estimated in terms of C_0, $C_0 = A_{eff} v_1$. Considering the pressure difference between the ventricle and the aorta is ΔP, by Bernoulli's equation with negligible h :

$$\Delta P = ½ \rho (v_1^2 - v_2^2)$$

v_2 is velocity inside the ventricle is neglected with respect to v_1, inside the aorta and hence:

$$2\Delta P A_{eff}^2 = \rho C_0^2 \rightarrow A_{eff} = C_0 \sqrt{(\rho / 2\Delta P)}$$

1.4. Velocity Measurement :

1.4.1 Venturi tube:

It consists of a conical constriction in a pipe from the full bore of cross-sectional area A_1, to a much narrower bore of area A_2 known as the throat of the Venturi tube and a gradual widening out again to the full bore as shown in fig.(1.5). A manometer is connected to measure the pressure difference. Applying Bernoulli's equation

with *h* constant along the same vertical plane:

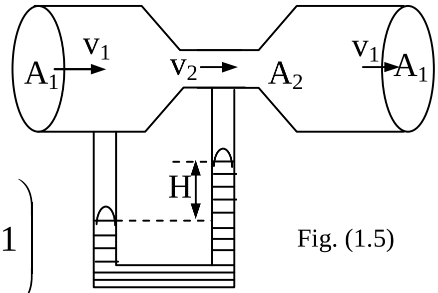


Fig. (1.5)

Illustration of the Venturi tube

$$\because A_1 v_1 = A_2 v_2$$

$$\therefore (P_1 - P_2) = \frac{1}{2}\rho v_2^2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)$$

$$\therefore \quad v_1^2 = \frac{2(P_1 - P_2)}{\rho\left((A_1/A_2)^2 - 1\right)}$$

In the manometer section: $P_1 - P_2 = (\rho' - \rho)gH$

$$\therefore v_1^2 = \frac{2(\rho' - \rho)gH}{\rho\left((A_1/A_2)^2 - 1\right)}$$

$$v_1 = k_1\sqrt{H/\rho} \qquad \text{.....(3.6)}$$

k_1 is the measurement constant and equal to $\dfrac{2\rho' g}{\left((A_1/A_2)^2 - 1\right)}$

$$Q = k_2\sqrt{\frac{H}{\rho}} \rightarrow k_2 = A_1 k_1$$

where ρ' is density of mercury , ρ is fluid density.

To calibrate the Venturi tube to measure the velocity and flow rate, *Q*, it is required to calculate the constants k_1 and k_2

1.4.2 Pitot Tube:

It consists of a central tube that opens through a narrow orifice in

the up-stream direction and is connected to the left arm of a manometer as shown in fig (1.6).

The surrounding outer tube, having an opening at C and another at A, is connected to the right arm of the manometer. The central stream line, at point A, enters the outer tube. The velocity at point B is considered zero as the fluid stops on entering the narrower tube.

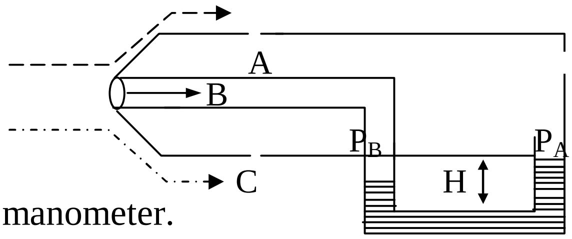


Fig (1.6)
Illustration of Pitot tube

$P_B - P_A = \rho' g\, h$

$$\because P_A + \frac{1}{2}\rho\, \mathrm{v}_\mathrm{A}^2 = P_B + \frac{1}{2}\rho\, \mathrm{v}_\mathrm{B}^2$$

$$\therefore P_B - P_A = \frac{1}{2}\rho\, \mathrm{v}_\mathrm{A}^2$$

$$\therefore v_A = \sqrt{\frac{2(P_B - P_A)}{\rho}} = \sqrt{\frac{2\rho'\mathrm{gH}}{\rho}} = \mathrm{k}_3 \sqrt{\mathrm{H}/\rho} \qquad \text{....(1.7)}$$

***Applications*:**

. Measuring blood velocity in arteries by introducing a Venturi tube of narrowed orifice into the artery using a large hygienic syringe.

. Measuring the quantity of blood flow using the Venturi tube.

. Designing injectors to introduce one or two fluids at a pressure less than the atmospheric pressure known as aspirators.

The injector, shown in fig.(1.7), is used to introduce oxygen at low pressure, in the thoracic cavity for those who suffer breathing difficulties.

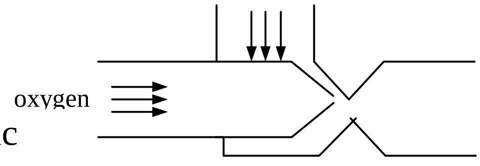


Fig. (1.7)

A typical aspirator

1.5 **<u>Flow of Real Fluids:</u>**

1.5.1 Viscosity:

When a real fluid moves along a tube, the fluid layer that is directly in contact with the wall of the tube is held by molecular forces and does not move at all. This stationary layer of fluid retards the layer next to it and the fluid velocity gradually increases as we move to the center of the tube. These drag forces are known as viscous forces, F, where: $F \ \alpha \ \frac{A_t v}{x}$

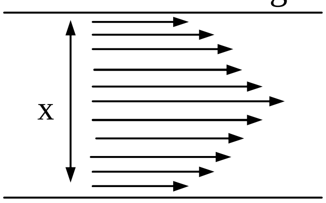


Fig. 1.8
Velocity profile of viscous fluid along a pipe

The proportionality constant dependso on the friction between the fluid layers, the coefficient of viscosity η.

$$F_v = A_t \ v/x$$

$$\therefore \ F_v/A_t = \eta \ \ v/x = \ -\eta \frac{dv}{dx} \qquad \text{...... (1.8)}$$

where A_t is the tangential area *(2πRl)*. Note that the quantity, F_v/A_t ,does not exert pressure on the walls as the viscous force is tangential to the area.

1.5.2. Velocity gradient:

It is defined as the change in velocity per unit thickness of fluid layer, *dv/dx*..

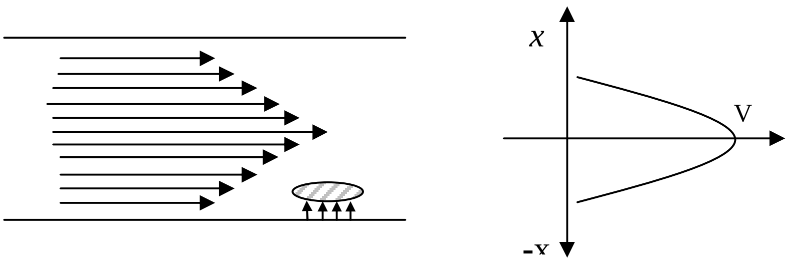


Fig. (1.9) Velocity profile of a viscous fluid

When fluid flows along a tube, the velocity is higher around the axis than near the walls, fig.(1.9) where the fluid is held back by its viscosity, so it is usually negative. This leads to an inverse pressure profile as pressure is higher at the walls and lower at the center. Thus the pressure on the side of an erythrocyte closer to the centre of the tube will be lower than the pressure on its side nearer to the wall. Hence, a red blood cell (RBC) is pushed towards the center of the tube if it comes nearer to the wall of a blood vessel. It is well known that erythrocytes are concentrated towards the center of blood vessels producing *axial streaming*. Axial streaming is an important aspect of blood flowing in blood vessels, where a layer of dilute plasma flows along the inner vascular wall while most of the erythrocytes flow along the axis with maximum velocity

1.5.3. Poiseuille's Equation:

The velocity of the fluid is zero next to the walls of the tube and increases gradually towards the center. At a distance *r* from the central line in any direction the speed will be the same i.e. the velocity gradient is axially symmetric. To analyze the total flow we consider the tube to consist of a series of concentric cylindrical shells, each moving at a different speed. They slide with respect to each other with friction equal to the viscous forces. The innermost cylinder moves with the highest speed, and the outer most is assumed stationary.

We start by studying the flow of a cylindrical shell of the fluid moving at a radius, *r*, with velocity *v(r)*, through a pipe of radius *R*. The shell is of radius *r* and thickness Δr as shown in fig (1.10)

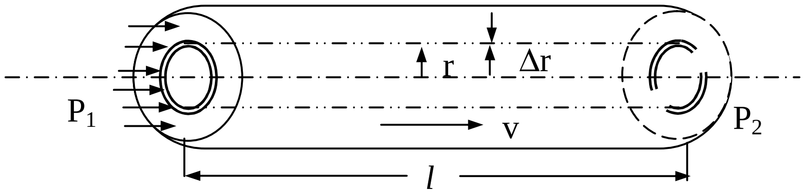


Fig. (1.10) A cylindrical layer of the fluid

The pressure difference *(P_1-P_2)* is what causes the motion of the fluid, so the driving force, F, is:

$$\therefore F = (P_1 - P_2) * \pi r^2$$

$$\frac{F}{A_t} = \frac{P_1 - P_2 * \pi r^2}{2\pi r \ell} = (P_1 - P_2) * \frac{r}{2\ell}$$

Across the thickness of the shell, Δr, the velocity change is Δv and the velocity gradient is equal to $(\frac{-dv}{dr})$. The negative sign indicates that the velocity decreases with the radius increases. Hence,

$$\left(\frac{-\Delta v}{\Delta r}\right) = \frac{(P_1 - P_2)r}{2\eta\ell}$$

$$\Delta v = -\frac{(P_1 - P_2)r}{2\eta\ell}\Delta r$$

$$dv = -\frac{(P_1 - P_2)}{2\eta\ell} r\, dr \qquad (1.9)$$

$$v = \frac{-(P_1 - P_2)}{2\eta\ell}\int r\, dr = -\left(\frac{P_1 - P_2}{2\eta\ell}\right)\frac{r^2}{2} + c$$

The constant **c** can be evaluated from the initial conditions of the problem. At the inner surface of the tube, where r = R, the fluid layer in contact with this surface is held by molecular forces and v = 0

$$\frac{R^2}{2} + C = 0 \qquad \therefore C = -R^2/2$$

$$\therefore v = \frac{P_1 - P_2}{4\eta \ell}\left(R^2 - r^2\right) \qquad (1.10)$$

To find a form that governs the flow rate, Q, and the pressure difference, consider a thin shell of the flow, shown in fig. (1.11), moving with velocity v, having radius r, the length that emerges from the end in a time t is vt and the contribution to the total volume is

$$\Delta V = v.t.\, 2\pi r\, \Delta r$$

$$\Delta V = \frac{P_1 - P_2}{4\eta \ell}\left(R^2 - r^2\right) t\ \ 2\pi r\, \Delta r$$

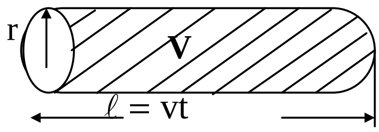


Fig. (3.11)

$$Q = \frac{dV}{dt} = B\left(R^2 - r^2\right) r\, dr$$

where $B = 2\pi \frac{P_1 - P_2}{4\eta \ell}$

To cover the whole volume of the tube we integrate from $r = 0$ to $r = R$

$$Q = B \int_0^R r\left(R^2 - r^2\right) dr = BR^2 \int_0^R r\ dt - B \int_0^R r^3\ dr$$

$$= B\left(R^2 \left.\frac{r^2}{2}\right|_0^R - \left.\frac{r^4}{4}\right|_0^R \right) = B\left(\frac{R^4}{2} - \frac{R^4}{4}\right) = \frac{B\ R^4}{4}$$

$$Q = 2\pi \ \frac{\left(P_1 - P_2\right)}{16\ \eta\ \ell} R^4$$

$$Q = \frac{\pi\ \left(P_1 - P_2\right) R^4}{8\ \eta\ \ell} \qquad ..... \ (1.11)$$

Eq.(1.11) is known as Poiseuille's equation that governs the flow rate, the pressure difference and the flow parameters. It describes the streamline flow to a

high-degree of accuracy, where η is constant across the tube and no particles of significant size are moving along with the fluid.

1.5.4. Application to blood flow analysis:

The flow rate is directly proportional to the quantity $\frac{P_1 - P_2}{\ell}$ which is known as the pressure gradient, $\frac{dP}{dx}$. It is directly proportional to the viscosity. Highly viscous fluids, for a given tube size and pressure difference, have lower flow rate than less viscous ones. Hence, to make the fluid move with the same flow rate, the pressure gradient should be increased.

The more viscous the blood becomes, the higher systolic pressure is required to keep its flow rate constant. The viscosity of the blood increases rapidly with the concentration of red blood cells (*RBC*). Thick blood with high ratio of red cell volume to total volume requires higher blood pressure pumped by the heart to keep the blood flow rate constant. This case is known as hematocrit.

In cases of shock the temperature of the body drops, thus the viscosity of the blood increases. According to Poiseulle's equation, the blood flow rate drops. The blood flow rate can return to normal if the body is warmed up. So the equation provides an explanation for the blood circulation disturbances that occur when the body temperature suddenly drops.

Example1.1:

If the viscosity coefficient of blood is 0.04 poise and increases to 0.045 poise. What is the percentage increase in the pressure gradient required to keep the blood flow rate constant?

Solution:

Change in $\eta = \frac{(0.045-0.04)}{0.004} * 100 = 12.5\%$

$\textit{Percentage change in}$ pressure grad. $= 12.5\%$

The most surprising aspect of Poiseulle's equation is the term R^4. The effect of the radius of the tube on the flow is illustrated in the following example.

Example1.2:

If a tube size was reduced a fifth of its original size, what change will occur on the flow rate?

Solution:

$$R_2 = R_1 - \frac{R_1}{5} = \frac{4}{5} R_1$$

$$\frac{dV_2}{dt} \alpha \left(R_2\right)^4 = \left(\frac{4}{5}\right)^4 R_1^4$$

Thus the flow rate, $Q = \frac{dV}{dt}$, decreases by $(4/5)^4 = 0.409$, about 40%

1.6 <u>Circuit Topology</u>:

It is often helpful to make an analogy between fluid flow and electric current flow. Poiseulle's equation can be analogous to Ohm's law governing the relation between the voltage, current and the resistance. The potential difference, voltage, is equivalent to the pressure difference, the current is equivalent to the flow rate and the resistance to the current is equivalent to the resistance to the flow, known as the peripheral resistance, R_p.

$$Q = \frac{dV}{dt} \leftrightarrow I$$

$$\Delta P \leftrightarrow \Delta V$$

$$Rp \leftrightarrow R \qquad \ldots(1.12)$$

$$where \qquad Rp = 8\,\eta l/\pi R^4$$

$$\therefore \ \Delta P/Q = Rp$$

R_p is the resistance of the vessel to the flow and depends on the viscosity coefficient, the length of the tube and inversely on the fourth power of the radius. It is clear that small changes in the radii of blood vessels can provide a sensitive regulation for the blood flow. Consequently, when R_p increases the quantity of the fluid passing per second through this blood vessel decreases and vice versa. Furthermore, we can readily derive rules for the resistance of the tubes in parallel or in series and show that they are the same rules for electric circuits.

The total peripheral resistance, TPR, is defined as the total resistance of the body to the blood flow through one complete cycle from the heart and back again. It can be calculated as the ratio of the ventricular pressure to the cardiac output, C_0.

1.7 <u>The physical significance of the Reynold's number</u> :

The type of flow which is a steady stream line flow is called laminar flow, where the liquid is considered to consist of concentric layers of hollow tubes of gradually decreasing radii. The motion of the liquid in this case is corresponding to that which occurs if the central tube is dragged along the axis. The slipping of the tubes, relative to one another, would result in a shear force between layers described earlier.

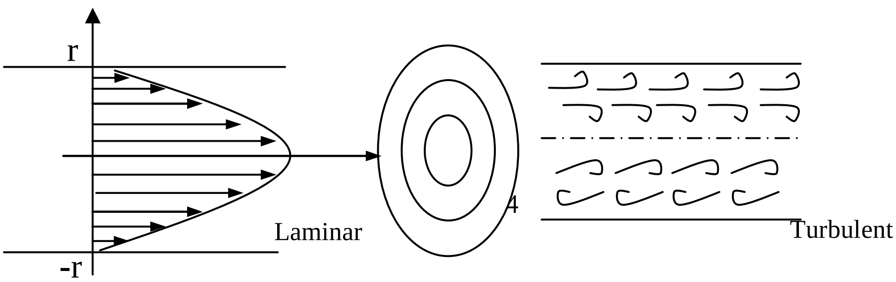


Fig. (1.12)

Laminar and turbulent flow in flow

The velocity profile as a result of this is known as laminar flow. If the rate of flow $\left(\frac{dV}{dt}\right)$ is increased beyond a certain limit, the flow becomes turbulent i.e. non-laminar as shown in fig. (1.12). The rate of change of momentum through a stream tube is equal to the inertial driving force and can given as $\boldsymbol{\rho A v^2}$. This force overcomes the shear stress due to viscosity action, $\boldsymbol{\eta v/R}$

Inertial force per unit cross sectional area, $\boldsymbol{\rho v^2}$, is thus compared with the viscous opposing force to produce what is called the Reynold's number.

Osborne Reynolds had analyzed the flow of liquids as discussed above and defined Reynolds number as:

$$N_{\mathrm{Re}} = \frac{\rho v R}{\eta} \qquad \text{….. (1.13)}$$

If $\boldsymbol{N_{Re}}$*>1000* the flow becomes turbulent. It is considered laminar as long as long as $\boldsymbol{N_R}$ is below that limit. For smooth surfaces of the tube the flow is usually laminar but as irregularities appear the Reynolds number increases. The ratio of the density to the viscosity of the fluid is the important issue.

Poiseille's equation is applicable for pure homogenous liquids only. There are some assumptions considered here that contradict the blood flow in the cardiovascular system.

Restrictions on applying Poisseulle's equation on blood:

* It doesn't hold for suspensions or mixtures of different liquids. Blood is an *anomalous liquid* that contains erythrocytes as RBCs, WBCs, platlets …etc.

The anomalous behavior in the case of blood is adjusted by the shape of the molecule in suspension. They are oval and rod like, so the internal forces of the liquid will tend to align them to the direction of flow, thus reducing the measured viscosity.

* In some blood vessels, as the pressure gradient increases the viscosity decreases ; $\frac{dP}{dx} \alpha \frac{1}{\eta}$, which contradicts the equation.

* The blood viscosity is not constant across the cross section of the blood vessel because of axial streaming. Besides, the viscosity differs from one part of the body to another and from one vessel to another.

Thus, blood is a *non-Newtonian* compressible fluid and the equation doesn't really apply to it. Yet it gives good approximation and can be applied for the overall circulation.

1.9 <u>Diffusion and Osmosis</u>

<u>1.9.1 Diffusion</u>:

Diffusion is the departure of the solute molecules from a solution of higher concentration to another of a lower concentration usually through a permeable membrane. Osmosis is the flow of the solvent molecules from a solution of lower concentration to another of higher concentration usually through a semi-permeable membrane.

Considering that diffusion occurs in one dimension, let's study the case of oxygen diffusing through a liquid that fills a long pipe of constant cross-sectional area.

The mass rate of diffusion passing from x_1 to x_2 can be given by *Fick's law:*

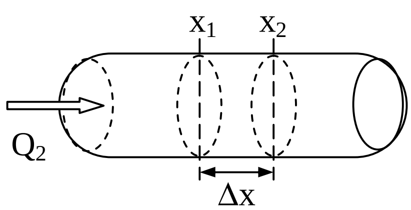


fig (1.15)
Unidimensional Diffusion of a homogeneous liquid

$$\frac{\Delta m}{\Delta t} = -DA\frac{\Delta c}{\Delta x}$$
$$\frac{dm}{dt} = -DA\frac{dc}{dx} \qquad \ldots \quad (1.15)$$

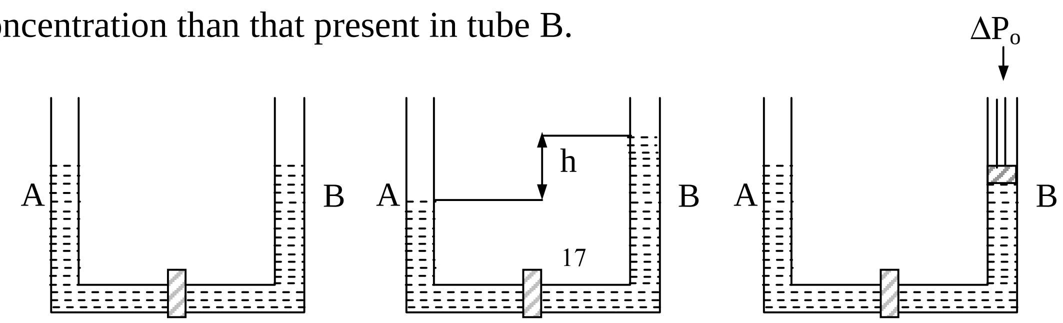
where $\Delta m = m_2 - m_1$, $\Delta C = C_2 - C_1$, C is the concentration , $A = c.s.a.$ and D is the diffusion constant. $D = \frac{kT}{6\pi a \eta}$ where a is the radius of the particle of solute, k is *Boltzmann's constant*, and T is the absolute temperature.

***1.9.2. Osmosis through a semi-permeable membrane*:**

OSMOSIS is the net movement of water across a selectively permeable membrane driven by a difference in solute concentrations on the two sides of the membrane. A selectively permeable membrane is one that allows unrestricted passage of water, but not solute molecules or ions.

Different concentrations of solute molecules leads to different concentrations of free water molecules on either side of the membrane. On the side of the membrane with higher free water concentration (i.e. a lower concentration of solute), more water molecules will strike the pores in the membrane in a give interval of time. More strikes equates to more molecules passing through the pores, which in turn results in net diffusion of water from the compartment with high concentration of free water to that with low concentration of free water.

A semi-permeable membrane allows the passage of the solvent's molecules only and prevents that of the solute. If we consider the two L-shaped tubes joined as shown in fig.(1.16.i). A semi-permeable membrane separates the two sugar solutions at the joining part. Tube A contains sugar solution of lower concentration than that present in tube B.



a b c

Fig. (1.16)

After a while, the level of water in tube B will rise as water passes through the membrane from A to B. The driving force of this flow is the difference in water potential between the two sides. This process of the solute's departure through a semi-permeable membrane is known as osmosis. Water potential of a phase, ψ, depends on two factors:

(a) The concentration of the solute; the higher the concentration the lower the potential becomes.

(b) The hydrostatic pressure on the phase; the higher the pressure the higher the potential becomes.

Hence, the water potential of each phase can be defined as:

$$\psi_1 = P_1 - RTC_1 \quad , \quad \psi_2 = P_2 - R\,T\,C_2$$

$$\psi_1 - \psi_2 = (P_1 - P_2) - R\,T\,(C_1 - C_2) \qquad \ldots\ (1.16)$$

The water flow, $J(kg/s)$ is defined as:

$$J = L_p\,(\psi_1 - \psi_2) \qquad \ldots\ (1.17)$$

When the pressure difference, ΔP equals $RT\Delta C$, or $\psi_1 = \psi_2$, the flow of water stops and $J = 0$. L_p is introduced here as the membrane porosity, its reciprocal indicates the membrane's resistance to the flow. It is worthwhile to notify that L_p is not constant for living tissue membranes.

$$J = L_p\,(\Delta P - RT\,\Delta C) \qquad \ldots\ (1.18)$$

where, ΔP is the difference in the pressure on both sides of the

semi-permeable membrane $(P_1 - P_2)$, ΔC is the difference in solution concentration across the membrane, $(C_1 – C_2)$.

OSMOSIS, the flow of water across a membrane in response to differing concentrations of solutes on either side, generates a pressure across the membrane called osmotic pressure. Osmotic pressure, ΔP_o , is defined as the hydrostatic pressure required to stop the flow of water, and thus, osmotic and hydrostatic pressures are, for all intents and purposes, equivalent, as shown in fig(1.16.c).

The membrane can be an artifical lipid bilayer, a plasma membrane or a layer of cells.

The osmotic pressure ΔP_o is found to be directly proportional to the concentration of the solute i.e. inversely proportional to the volume of the solution. It is also proportional to the temperature.

The osmotic pressure P of a dilute solution is generally approximated by the following:

$$P = RT\,(C1 + C2 + .. + Cn)$$

where R is the gas constant (8.314j/kgmole or 0.082 liter-atmosphere/degree-mole), T is the absolute temperature, and C1 ... Cn are the molar concentrations of all solutes (ions and molecules). Thus

$$\Delta P_O = RT\Delta C \qquad \ldots(1.19)$$

Hence equation (1.16) can be written as:

$$J = L_p\,(\Delta P - \Delta P_0) \qquad \ldots(1.20)$$

The osmolarity of a solution is an index of the number of particles present in one litre of the solution. Concentrations are given either as molarity ; grams/liter or measured in milli-osmoles per liter of water. To compare the osmolarity of one solution to another the terms hyperosmotic, isosmotic or hyposmotic are used to

indicate that one solution contains higher, the same or lower concentration of dissolved solute than the other respectively.

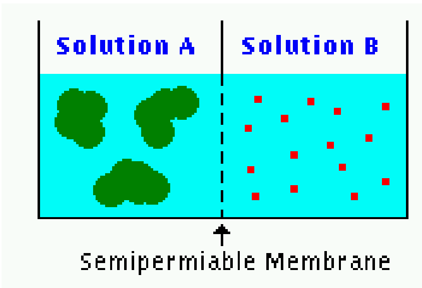
***1.9.4 Osmosis in biological processes*:**

All living cells have semi-permeable membranes that permit selective diffusion of the molecules that the cells need. Large quantities of water molecules constantly move across cell membranes by simple diffusion, but, in general, net movement of water into or out of cells is negligible.

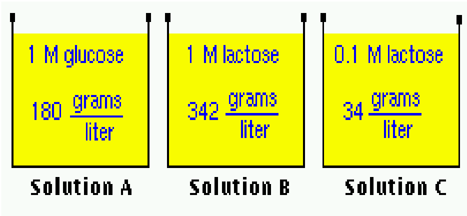
For example, it has been estimated that an amount of water equivalent to roughly 250 times the volume of the cell diffuses across the red blood cell membrane *every second*; the cell doesn't lose or gain water because equal amounts go in and out. There are, however, many cases in which net flow of water occurs across cell membranes and sheets of cells. An example of great importance is the secretion of and absorption of water in small intestine. In such situations, water still moves across membranes by simple diffusion, but the process is followed by osmosis to retain the amount of water diffused inside.

"Water flows from the solution with the lower solute concentration into the solution with higher solute concentration", means that water flows in response to differences in molarity across a membrane. *The size of the solute particles does not influence osmosis*. Equilibrium is reached once sufficient water has moved to equalize the solute concentration on both sides of the membrane, and at that point, net flow of water ceases. Here is a simple example to illustrate these principles:

Example1.3: Two containers of equal volume are separated by a membrane that allows free passage of water, but totally restricts passage of solute molecules. Solution A has 3 molecules of the protein albumin (molecular weight 66,000) and Solution B contains 15 molecules of glucose (molecular weight 180). Into which compartment will water flow, or will there be no net movement of water?

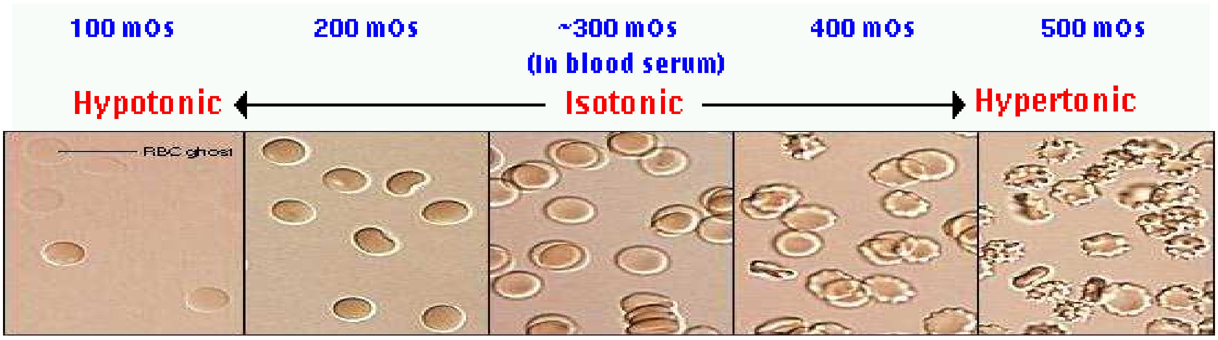


Example1.4: In the tubes below, Solutions A and B are Isotonic (with each other), Solutions A and B are hypertonic compared to Solution C, and Solution C is hypotonic relative to Solutions A and B.



1 osmolar solution contains 1 mole of osmotically-active particles (molecules and ions) per liter. **The classic demonstration of osmosis and osmotic pressure is to immerse red blood cells in solutions of varying osmolarity and watch what happens.** Blood serum is isotonic with respect to the cytoplasm, and red cells in that solution assume the shape of a biconcave disk. To prepare the images shown below, red cells were suspended in three types of solutions: **Isotonic** - the cells were diluted in serum: Note the beautiful biconcave shape of the cells as they circulate in blood. **Hypotonic** - the cells in serum were diluted in water: At 200 milliosmols (mOs), the cells are visibly swollen and have lost their biconcave shape, and at 100 mOs, most have swollen so much that they have ruptured, leaving what are called red blood cell ghosts. In a hypotonic solution, water rushes into cells. **Hypertonic.** A concentrated solution of NaCl was mixed with the cells and serum to increase osmolarity:

Example1.5: At 400 mOs and especially at 500 mOs, water has flowed out of the cells, causing them to collapse and assume the spiky appearance. Predict what would happen if you mixed sufficient water with the 500 mOs sample shown below to reduce its osmolarity to about 300 mOs.



***1.9.5 Osmosis in biological processes, Dialysis*:**

The waste products of metabolism are removed from the blood by osmosis in the kidneys by a process known as dialysis. When the blood flows into the kidneys it enters a cluster of capillaries known as glomerulus. A kidney contains about 10^6 nephrons, each is 50 mm long and 20-50 μm wide. A nephron, shown in fig. (1.17), has one closed end and opens into a collecting duct. The total length of these tubules in the two kidneys is about 70 miles.

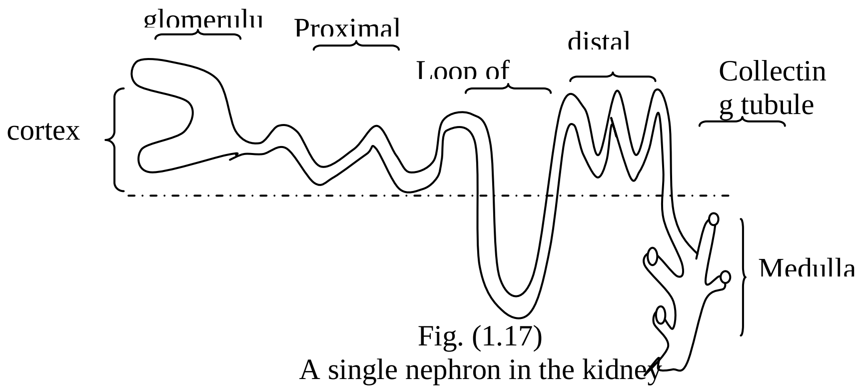


Fig. (1.17)
A single nephron in the kidney

The glomerulus has both a semi-permeable and filtering membrane that bars the passage of the blood colloids and permits only waste materials. The process of filtration requires a hydrostatic pressure to force the solvent through the membrane, as opposed to the process of osmosis. The separation of solvent and solute actually generates an osmotic pressure in the opposite direction where blood becomes hyper-osmotic than the solution inside the glomerulus. The pressure required to cause the flow through the tubules is 40 *torr* and osmotic pressure of the blood colloids is 30 *torr*. Therefore the total minimum pressure required for the kidneys to function is 70 *torr*. The solution that diffuses to the kidney tubules by osmosis following the passage of waste substances by filtration is finally excreted as urine.

The tonicity of urine is the measure of its *osmolarity* w.r.t. the blood plasma. If it is hypertonic, isotonic or hypotonic, then its osmolarity is higher than, equal to, or lower than that of plasma respectively. The tonicity of plasma is approximately *300 m osmole/kg* H_2O.

If one would try the blood-water comparision, it is found that "physiological saline" , present on the blood side, generates sufficient osmotic pressure to support a column roughly 70 meters high!

Example 1.6: Determine the pressure generated when 8.5 grams/liter NaCl (0.145 M) is on one side and water on the other - this is pretty close to the situation of having blood on one side and water on the other. **Assumed temperature is 20C or 293K, R=**0.082 lit.atm/mole.K

Solution: The calculation performed, without accounting for effective concentration, is: $\Delta P_o = RT\Delta C$.

Hint: NaCl: molecular weight 58.44; dissociates in solution into sodium and chloride ions **Sucrose** (table sugar): molecular weight = 342.3; does not dissociate **Albumin**: molecular weight approximately 66,000; a single protein constitutes 3% of plasma.